

Name: Solutions / Answers

Directions for # 1-8: For each rational function:

1. Identify an equation for each vertical asymptote, if any exist.
2. Identify an equation for each horizontal asymptote, if any exist.
3. Identify the coordinates of any y-intercept or x-intercepts, if any exist.
4. Identify any value of x for which the graph has a hole
5. Draw a neat and accurate graph of the function

1.  $f(x) = \frac{6}{x-3}$

VA:  $x = 3$

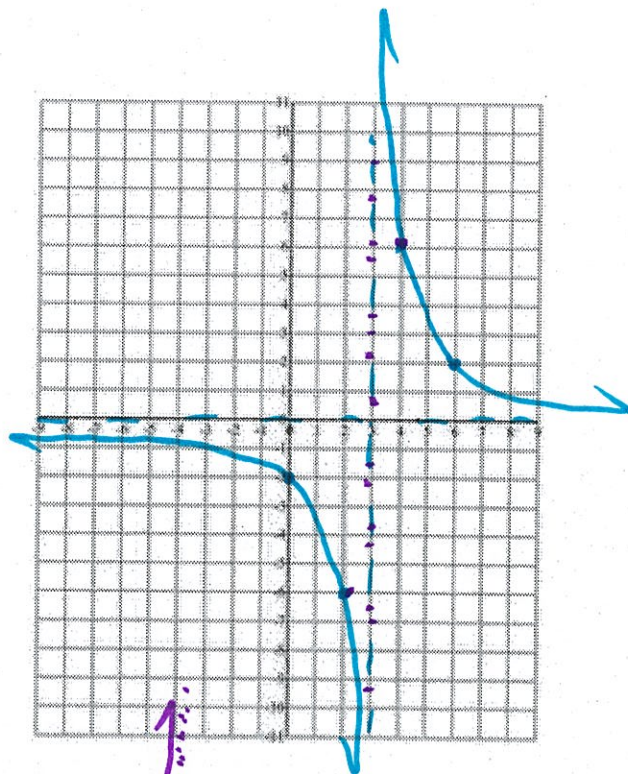
HA:  $y = 0$

X-Intercept(s): *None*

Y-Intercept:  $(0, -2)$

X-Value of Hole: *None*

$$\begin{array}{c|c} X & Y \\ \hline 2 & -6 \end{array}$$



2.  $f(x) = \frac{2x}{x+4}$

VA:  $x = -4$

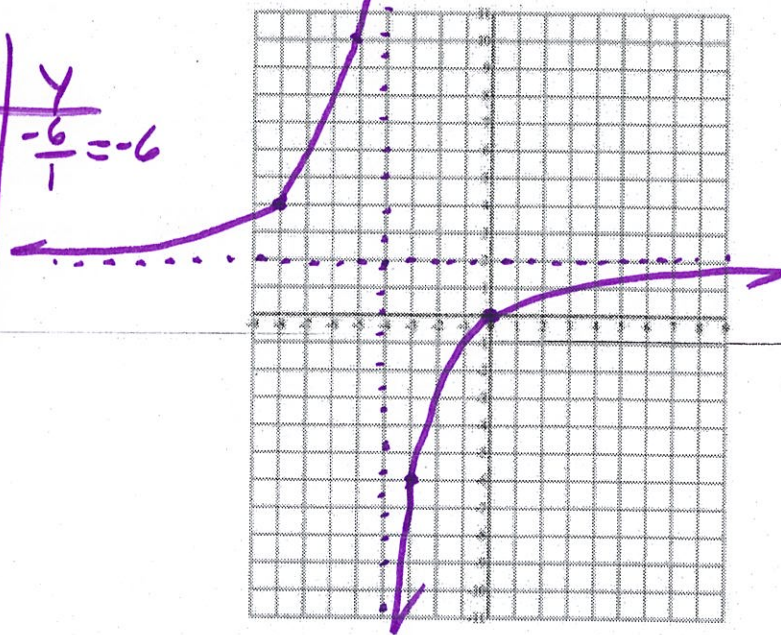
HA:  $y = 2$

X-Intercept(s):  $(0, 0)$

Y-Intercept:  $(0, 0)$

X-Value of Hole: *None*

$$\begin{array}{c|c} X & Y \\ \hline -3 & -\frac{6}{1} = -6 \end{array}$$





3.  $f(x) = \frac{x+4}{x-2}$

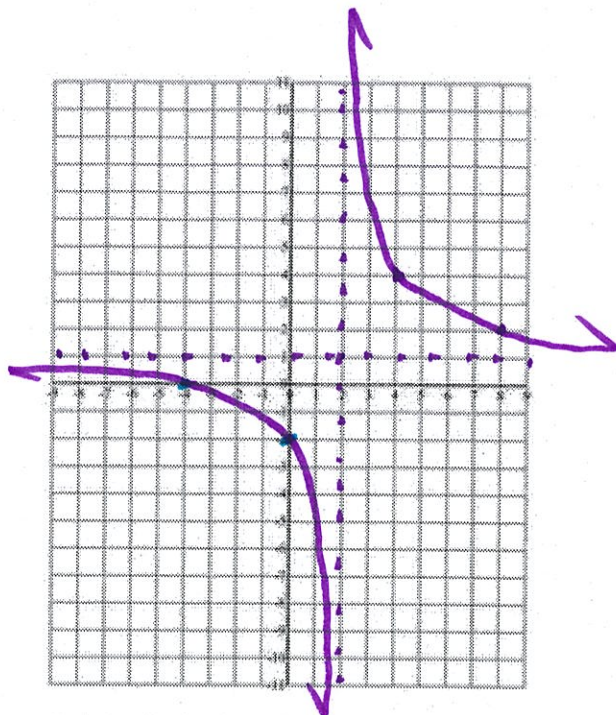
VA:  $x = 2$

HA:  $y = 1$

X-Intercept(s):  $(-4, 0)$

Y-Intercept:  $(0, -2)$

X-Value of Hole: None



4.  $f(x) = \frac{x^2+2x-3}{x-1}$  or  $f(x) = \frac{(x-1)(x+3)}{x-1}$

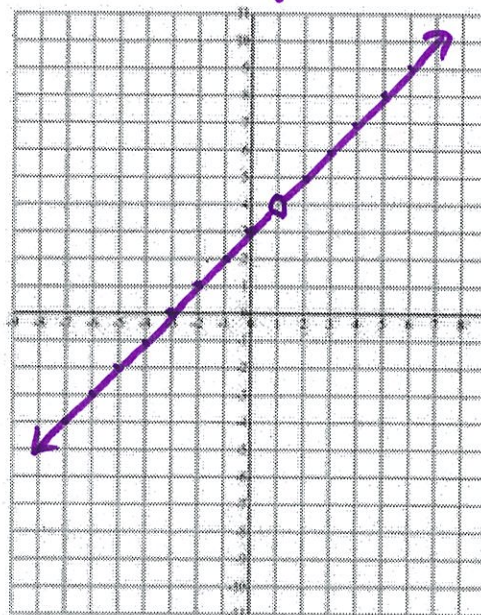
VA: None

HA: None

X-Intercept(s):  $(-3, 0)$

Y-Intercept:  $(0, 3)$

X-Value of Hole:  $x = 1$



5.  $f(x) = \frac{x-1}{x^2+2x-3}$  or  $f(x) = \frac{x-1}{(x-1)(x+3)}$

VA:  $x = -3$

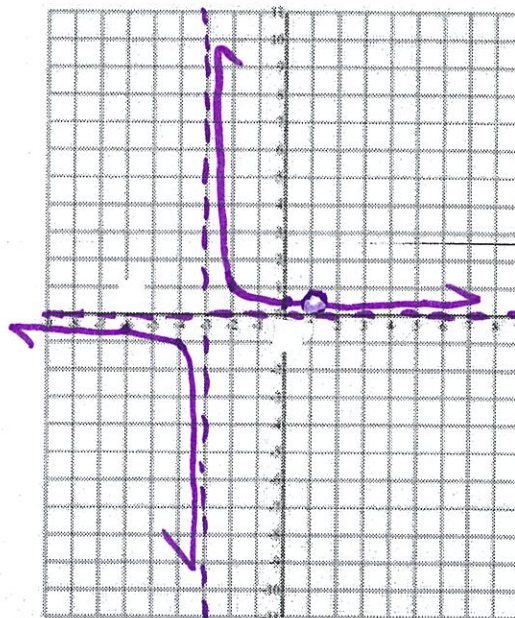
HA:  $y = 0$

X-Intercept(s): None

Y-Intercept:  $(0, \frac{1}{3})$

X-Value of Hole:  $x = 1$

$x$	$y$
$-2$	$1$





6.  $f(x) = \frac{x-4}{x^2-4}$  or  $f(x) = \frac{x-4}{(x+2)(x-2)}$

VA:  $x = -2$  &  $x = 2$

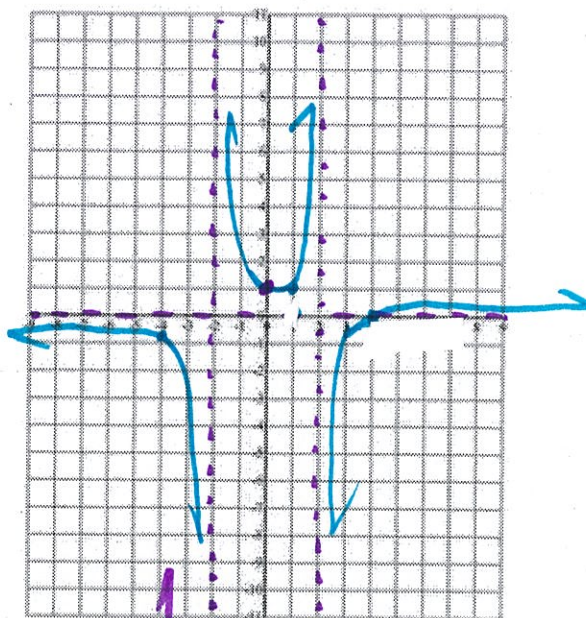
HA:  $y = 0$  x-axis

X-Intercept(s):  $(4, 0)$

Y-Intercept:  $(0, 1)$

X-Value of Hole: None

X	Y
1	1
-4	$\frac{-8}{12} = -\frac{2}{3}$
6	$\frac{2}{32}$
3	$\frac{-1}{5}$



7.  $f(x) = \frac{3x^2-3}{x^2+2x-3}$  or  $f(x) = \frac{3(x+1)(x-1)}{(x-1)(x+3)}$

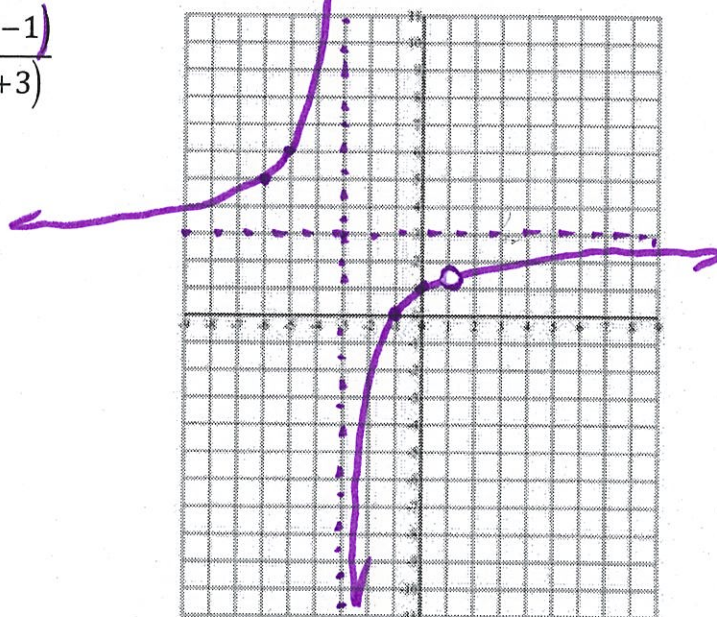
VA:  $x = -3$

HA:  $y = 3$

X-Intercept(s):  $(-1, 0)$

Y-Intercept:  $(0, 1)$

X-Value of Hole:  $x = 1$



8.  $f(x) = \frac{6x^2}{2x^2-8} = \frac{6x^2}{2(x-2)(x+2)}$

VA:  $x = 2$  &  $x = -2$

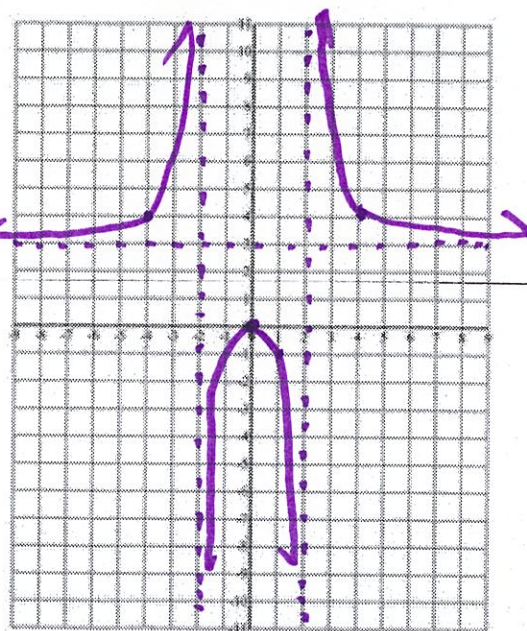
HA:  $y = 3$

X-Intercept(s):  $(0, 0)$

Y-Intercept:  $(0, 0)$

X-Value of Hole: None

X	Y
1	$\frac{6}{-6} = -1$
4	$\frac{96}{24} = 4$
-4	$\frac{96}{24} = 4$



## MULTIPLE CHOICE

9. The rational function  $f(x) = \frac{x^2 - 9}{x^2 + 8x - 33}$  has the following domain:

a.  $D: \mathbb{R}, x \neq -3, x \neq 11$

b.  $D: \mathbb{R}, x \neq -3, x \neq -11$

c.  $D: \mathbb{R}, x \neq 3, x \neq 11$

☒ d.  $D: \mathbb{R}, x \neq 3, x \neq -11$

e.  $D: \mathbb{R}$

$$(x+11)(x-3)$$

10. The rational function  $f(x) = \frac{x^2 + 2x - 15}{x^2 + 100}$  has the following domain:

a.  $D: \mathbb{R}, x \neq 10, x \neq -10$

b.  $D: \mathbb{R}, x \neq -10$

c.  $D: \mathbb{R}, x \neq 10$

d.  $D: \mathbb{R}, x \neq 10, x \neq -10$

☒ e.  $D: \mathbb{R}$

11. The rational function  $f(x) = \frac{5}{x-9}$  has a vertical asymptote with equation:

a.  $y = 5$

☒ b.  $x = 9$

c.  $y = 0$

d.  $x = -9$

e. No vertical asymptotes

12. The rational function  $f(x) = \frac{19}{x+6}$  has a horizontal asymptote with equation:

a.  $y = 19$

b.  $x = 0$

c.  $y = -6$

☒ d.  $y = 0$

e. There is no horizontal asymptote

13. The rational function  $f(x) = \frac{2}{x^2 - 9}$  has a vertical asymptote with equation:

a.  $y = -3$  &  $y = 3$

b.  $x = 3$

c.  $x = 2$

☒ d.  $x = -3$  &  $x = 3$

e. None of the above

$$f(x) = \frac{2}{(x-3)(x+3)}$$



14. The rational function  $f(x) = \frac{4x^3 + 9x}{2x^3 + x^2 - 5}$  has a horizontal asymptote with equation:

- a.  $x = 2$
- ☒ b.  $y = 2$
- c.  $y = 4$
- d.  $y = 0$
- e. None of the above

$$y = \frac{4}{2} = 2$$

15. The rational function  $f(x) = \frac{x-4}{x^2-16}$  has a hole at:

- a.  $x = -4$
- b.  $x = \frac{1}{4}$
- ☒ c.  $x = 4$
- d.  $x = 16$
- e. There is no hole

$$f(x) = \frac{x-4}{(x-4)(x+4)}$$

$x-4$  is common to top and bottom

16. The rational function  $f(x) = \frac{x+10}{x-2}$  has a y-intercept at:

- a.  $y = -10$
- b.  $y = 0$
- c.  $x = 2$
- ☒ d.  $y = -5$
- e. There is no y-intercept

$$\text{Let } x = 0$$

17. The rational function  $f(x) = \frac{x+6}{x^2-2}$  has an x-intercept at:

- a.  $x = 6$
- b.  $x = -3$
- ☒ c.  $x = -6$
- d.  $x = 2$
- e. There is no x-intercept

$$\begin{aligned} x+6 &= 0 \\ x &= -6 \end{aligned}$$

18. The rational function  $f(x) = \frac{x-5}{x^2-25}$  has an x-intercept at:

- a.  $x = 5$
- b.  $y = \frac{1}{5}$
- c.  $x = -5$
- d.  $x = 0$
- ☒ e. There is no x-intercept

$$x-5 \neq 0 \quad f(x) = \frac{x-5}{(x-5)(x+5)}$$

hole at  $x = 5$   
No x-intercept at  $x = 5$

19. The rational function  $f(x) = \frac{x^2 - 16}{x^2 - 6x + 8}$  has a vertical asymptote with equation:

- a.  $x = 4$  &  $x = 2$
- b.  $x = 4$  only
- ☒ c.  $x = 2$  only
- d.  $x = -2$  only
- e. There is no vertical asymptote

$$f(x) = \frac{(x-4)(x+4)}{(x-2)(x-4)}$$

hole at  $x = 4$

20. The rational function  $f(x) = \frac{4}{x^2 + 4}$  has a vertical asymptote with equation:

- a.  $y = -2$  &  $y = 2$
- b.  $y = 4$
- c.  $x = 4$
- d.  $x = -2$  &  $x = 2$
- ☒ e. There is no vertical asymptote

$$x^2 + 4 \neq 0 \text{ ever}$$

21. The rational function  $f(x) = \frac{6x + 9}{2x^2 - 4x + 11}$  has a horizontal asymptote with equation:

- ☒ a.  $y = 0$
- b.  $y = 3$
- c.  $y = 6$  &  $y = 2$
- d.  $x = \frac{9}{11}$
- e. There is no horizontal asymptote

22. The rational function  $f(x) = \frac{x^2 + 5x - 14}{x^2 + 10x + 21}$  has a hole at:

- a.  $x = -3$  &  $x = -7$
- b.  $x = 3$  &  $x = 7$
- c.  $x = -3$  only
- ☒ d.  $x = -7$  only
- e. There is no hole

$$f(x) = \frac{(x+7)(x-2)}{(x+7)(x+3)}$$

$x+7$  is common to the top and bottom

Directions: Write an equation for each rational function shown below

23.

VA:  $x = 3$

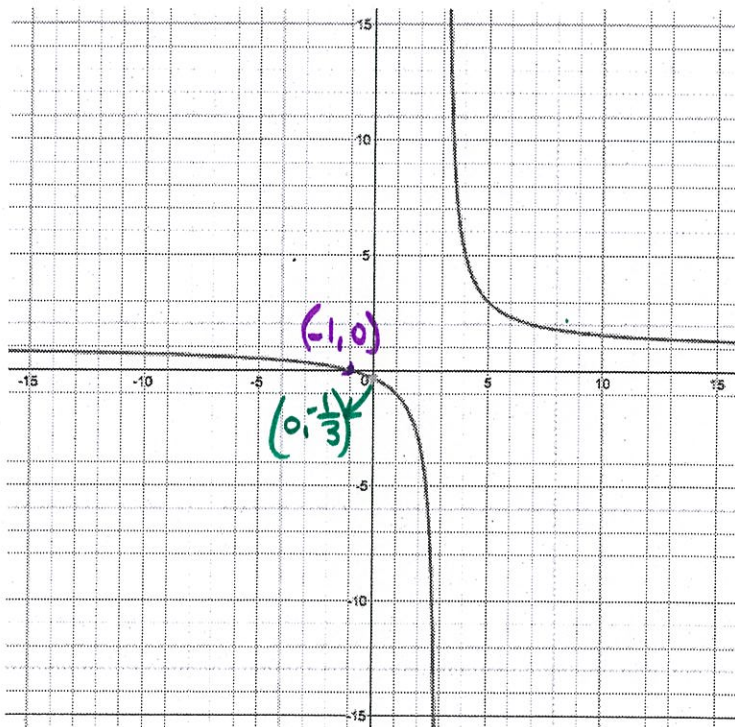
HA:  $y = 1$

X-Intercept(s):  $(-1, 0)$

Y-Intercept:  $(0, -\frac{1}{3})$

X-Value of Hole: *None*

$$y = \frac{x+1}{x-3}$$



24.

VA:  $x = -3$

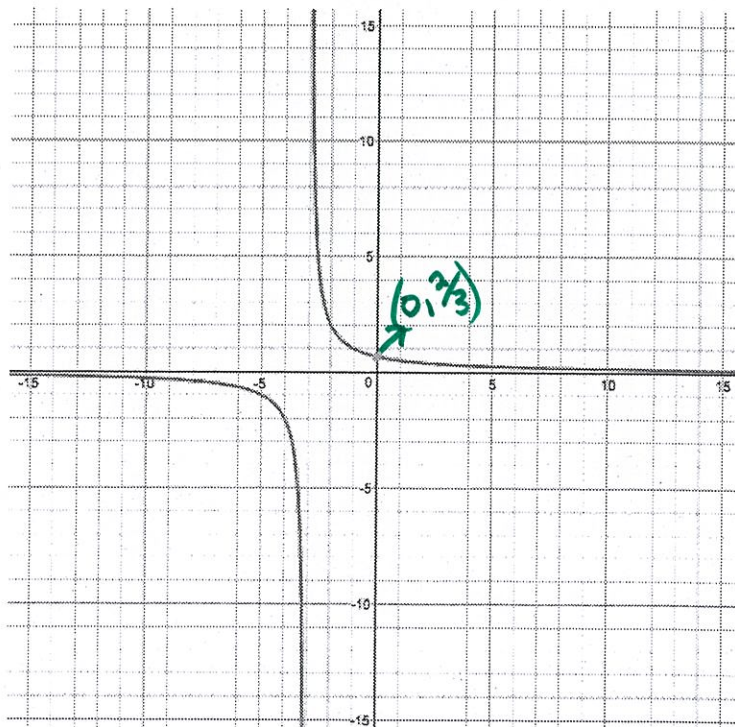
HA:  $y = 0$  x-axis

X-Intercept(s): *None*

Y-Intercept:  $(0, \frac{2}{3})$

X-Value of Hole: *None*

$$y = \frac{2}{x+3}$$





25.

VA:  $x=1$

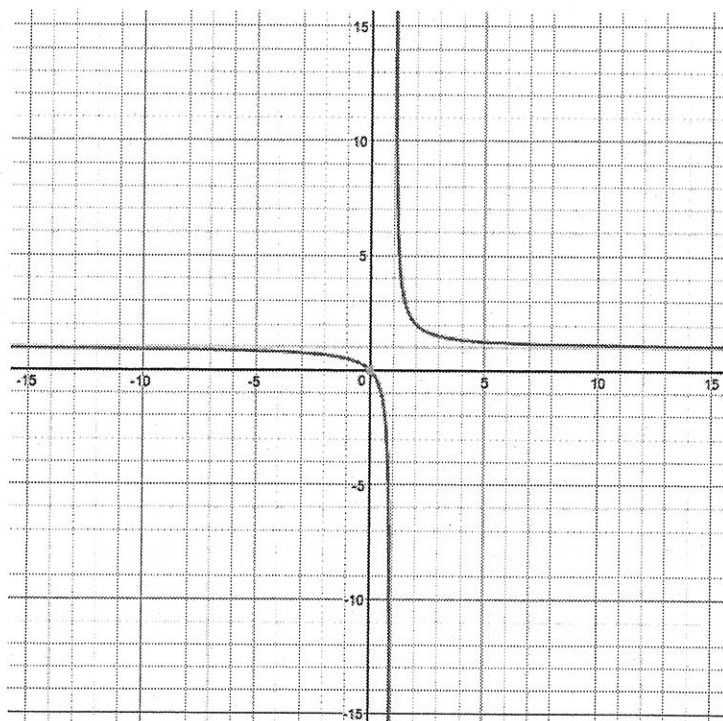
HA:  $y=1$

X-Intercept(s):  $(0,0)$

Y-Intercept:  $(0,0)$

X-Value of Hole: None

$$y = \frac{x}{x-1}$$



26.

VA:  $x=-1$

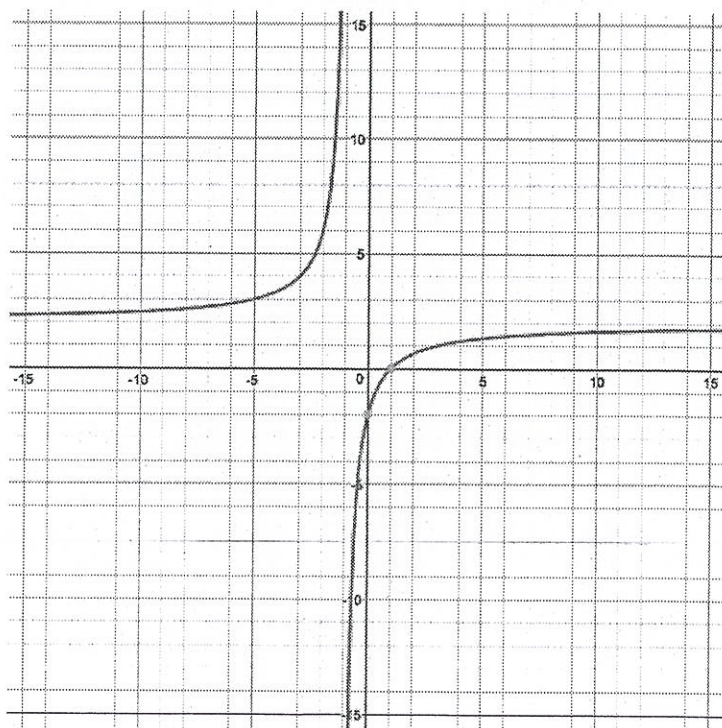
HA:  $y=2$

X-Intercept(s):  $(1,0)$

Y-Intercept:  $(0,-2)$

X-Value of Hole: None

$$y = \frac{2(x-1)}{x+1}$$





27.

VA:  $x=1$

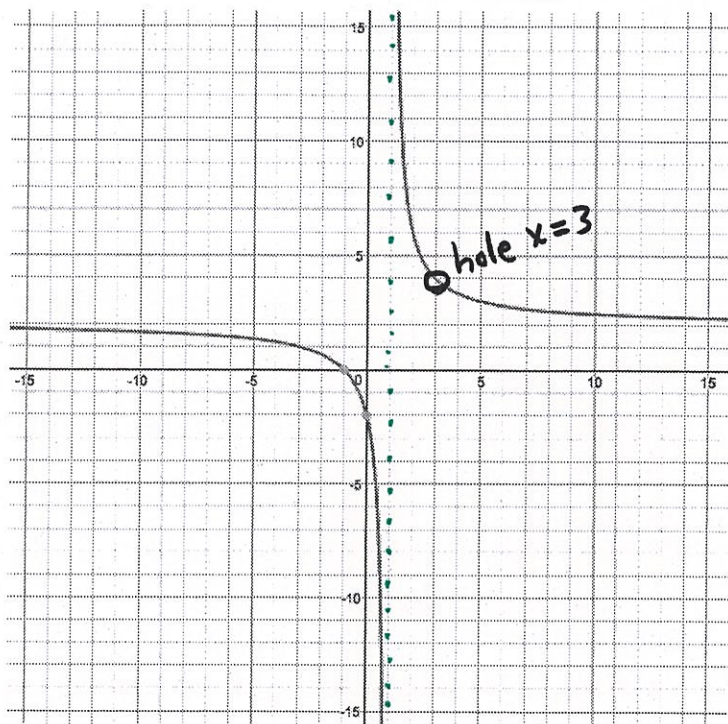
HA:  $y=2$

X-Intercept(s):  $(-1, 0)$

Y-Intercept:  $(0, -2)$

X-Value of Hole: at  $x=3$

$$y = \frac{2(x-3)(x+1)}{(x-3)(x-1)}$$



28.

VA: None

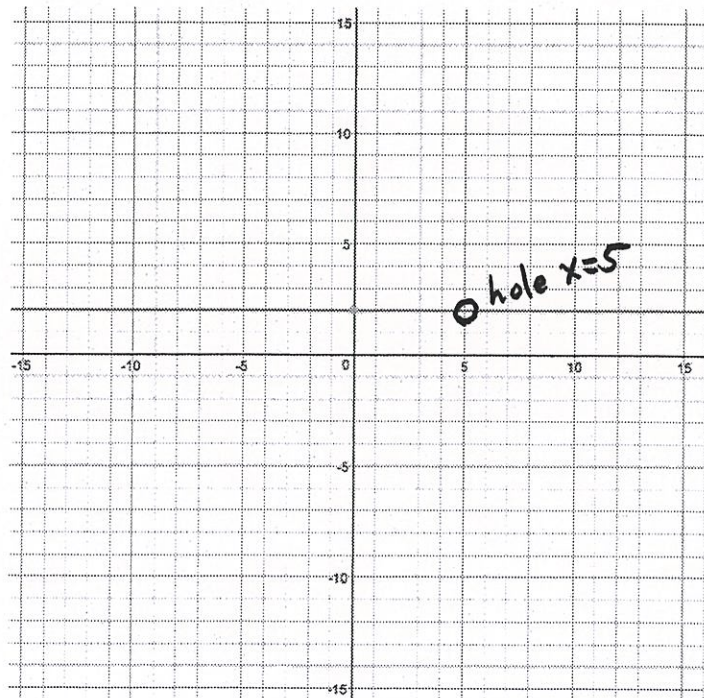
HA:  $y=2$

X-Intercept(s): None

Y-Intercept:  $(0, 2)$

X-Value of Hole:  $x=5$

$$y = \frac{2(x-5)}{(x-5)}$$



29.

VA:  $x = 2$

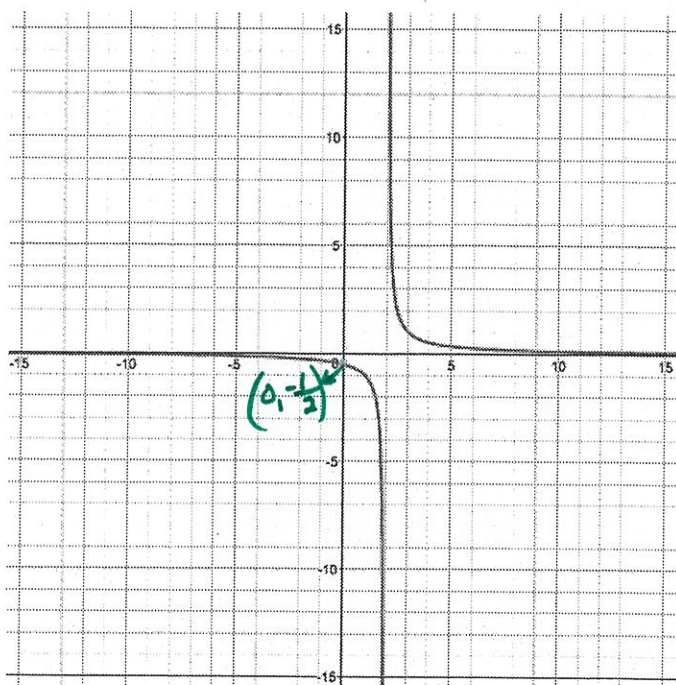
HA:  $y = 0$  x-axis

X-Intercept(s): None

Y-Intercept:  $(0, -\frac{1}{2})$

X-Value of Hole: No hole

$$y = \frac{1}{x-2}$$



30.

$$y = \frac{(x+3)(x-2)}{x+3}$$

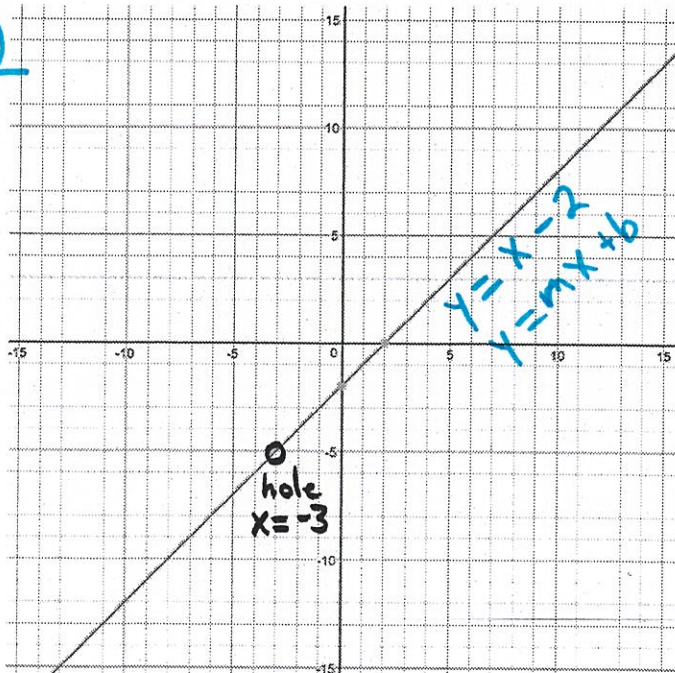
VA: None

HA: None

X-Intercept(s):  $(2, 0)$

Y-Intercept:  $(0, -2)$

X-Value of Hole:  $x = -3$





31.

VA:  $x = -3$  &  $x = 2$

HA:  $y = 0$

X-Intercept(s):  $(-6, 0)$

Y-Intercept:  $(0, -1)$

X-Value of Hole: *None*

$$y = \frac{x+6}{(x+3)(x-2)}$$

